

Contents: **Lessons 1 to 24**

		Page
Lesson 1	Number Systems and Introduction	4
Lesson 2	Functions	10
Lesson 3	Algebra	20
Lesson 4	Algebra II	28
Lesson 5	Algebra III	36
Lesson 6	Algebra IV	46
Lesson 7	Geometry I	54
Lesson 8	Geometry II	64
Lesson 9	Trigonometry	70
Lesson 10	Coord Geo of the Line	80
Lesson 11	Statistics I	92
Lesson 12	Probability I	102
Lesson 13	Statistics II	110
Lesson 14	Statistics III & Probability II	118
Lesson 15	Calculus I	128
Lesson 16	Calculus II	136
Lesson 17	Calculus III	146
Lesson 18	Calculus III	158
Lesson 19	Financial Maths I	166
Lesson 20	Financial Maths II	174
Lesson 21	Induction & Logs	184
Lesson 22	Complex Numbers I	194
Lesson 23	Complex Numbers II	204
Lesson 24	Exam Preparation	212

Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 1: Number Systems and Introduction

1.1 Learning Intentions

After this week's lesson you will be able to;

- Differentiate between the various number systems (Natural, Integers etc.)
- Construct line segments with lengths of irrational value
- Represent numbers in scientific notation
- Represent a number correct to a certain number of significant figures

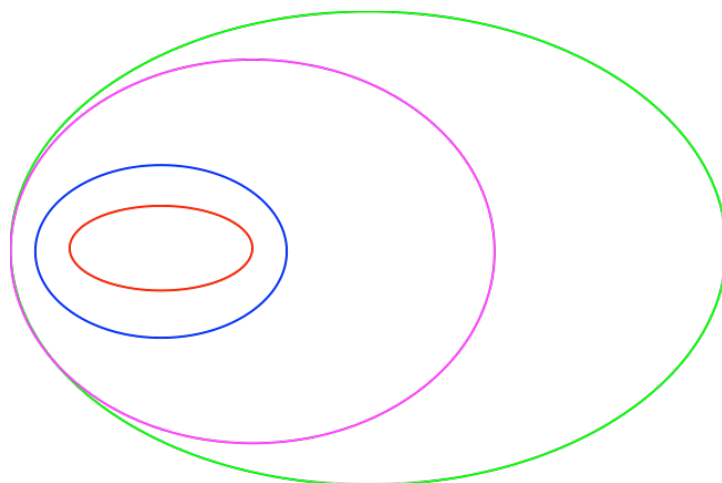
1.2 Specification

Student learn about	Student working at OL should be able to	In addition, students working at HL should be able to
3.1 Numer systems	<ul style="list-style-type: none">– recognise irrational numbers and appreciate that $\mathbf{R} \neq \mathbf{Q}$– work with irrational numbers– revisit the operations of addition, multiplication, subtraction and division in the following domains:<ul style="list-style-type: none">▶ \mathbf{N} of natural numbers▶ \mathbf{Z} of integers▶ \mathbf{Q} of rational numbers▶ \mathbf{R} of real numbers and represent these numbers on a number line	<ul style="list-style-type: none">– geometrically construct $\sqrt{2}$ and $\sqrt{3}$– prove that $\sqrt{}$ is not rational

1.3 Chief Examiner's Report

The question often contains clues regarding the nature of the answer. For instance, if it asks to find the value of x , there should only be one answer, while if it asks to find the values of x , one might expect more than one. Similarly, attention should be paid to whether the question indicates that the answer is a natural number, integer, real number, etc.

1.4 Number Systems



Natural Numbers

Rational Numbers

Irrational Numbers

Integers

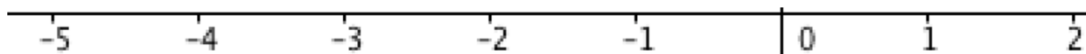
Real Numbers

Complex Numbers

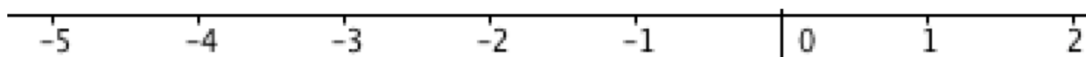
1.5 Placing Numbers on the number line

The most popular place for these symbols is in representing a solution on a number line, particularly in inequalities.

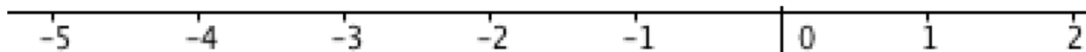
$$x < 2, \text{ where } x \in \mathbb{N}$$



$$x < 2, \text{ where } x \in \mathbb{Z}$$

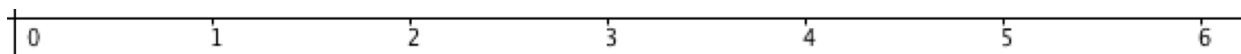


$$x < 2, \text{ where } x \in \mathbb{R}$$

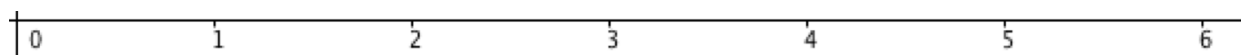


In some cases, we may be told that the solution can be greater/less than or equal to a particular value. Let's have a look at such a situation

$$x \geq 2 \text{ or } x > 2 \text{ if } x \in \mathbb{N}$$



$$x \geq 2 \text{ or } x > 2 \text{ if } x \in \mathbb{R}$$



1.6 Prime Numbers

Prime numbers are defined as the numbers that have two distinct factors, the number and itself.

Your Attempt

3 5 9 7 1

Correct Answers

3 5 9 7 1

A number that is not a prime number, i.e. has more than two factors, is known as a composite number.

Based on a theorem by the Greek mathematician, Euclid, every natural number greater than 2 is either prime or a product of primes.

What this means is that we can break up a term into the prime factors that multiply to give that term. For example:

240

1.7 Irrational Numbers

These are numbers that cannot be written as a fraction using integer. The most common example of these numbers is π which is **approximately** equal to 3.14. However, the exact value of pi cannot be expressed as a fraction. They can be represented by the following symbol $\mathbb{R} \setminus \mathbb{Q}$

Other examples of irrational numbers include:

$\sqrt{2}$ and $\sqrt{3}$ both of which we will look at next.

Constructing: $\sqrt{2}$:

To get the most out of this construction, be sure to have both the video and notes in front of you when attempting to follow it.

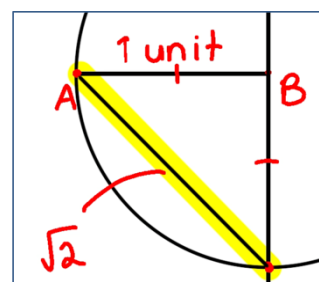
Step 1: Draw a line segment that is 1 unit in length (cm, inch etc.)

Step 2: Using either a ruler or set square, draw a line that is perpendicular to $[AB]$ and passes through the point B.

Step 3: Using your compass and a radius $[AB]$ draw a circle.

Step 4: Join A to one of the points of intersection between the circle and the perpendicular line to form a right-angle triangle.

Proof of $\sqrt{2}$:



Constructing $\sqrt{3}$:

Step 1: Draw a line segment that is 1 unit in length (cm, inch etc.)

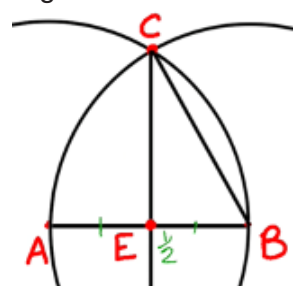
Step 2: Using your compass and a radius $[AB]$ draw a circle using the point A as the centre of the circle.

Step 3: Repeat step 2 but use the point B as the centre to create a second circle.

Step 4: Label the two points where the two circles intersect as C and D. Then connect C to D using a straight edge to create the line segment CD

Step 5: $[CD]$ bisects $[AB]$, label the point where these two lines intersect as E. Thus creating $[EB]$ and $[EA]$ both of length $\frac{1}{2}$

Step 6: We then join the point C to the point B to create our final line segment.



Proof of $\sqrt{3}$:

1.8 Significant Figures

As mentioned in the video, significant figures are quite similar to decimal places in many ways, however there is a difference. Follow the video clip and fill in the table below:

3 Decimal Places 3 Significant Figures

3.67834

On first inspection these concepts look identical, however if we use a smaller number and carryout the same process, we can see there is a difference, follow the video again and complete the table:

3 Decimal Places 3 Significant Figures

0.004589

An important thing to remember is that:

A zero at the start of a number is not significant

1.9 Scientific Notation

This style of number is used to show both extremely large values, and extremely small values. It works on the idea of orders of magnitude. For example, 100 is one order of magnitude greater than 10. To go **up** in orders of magnitude we simply **multiply by 10** and to go **down** in orders of magnitude, we **divide by 10**. See slide 22 for an example of a large number and slide 23 for an example of a small number.

Moon Example:

The moon is 384000 km from the earth which can be converted into scientific notation by firstly shifting the decimal point between the 1st and 2nd **significant** digit and then we multiply by 10 the appropriate number of times to illustrate the difference in orders of magnitude.

Standard Form: 384400.0 km

Scientific Notation: 3.8×10^5 km

Water Molecule Example:

A molecule of water has a mass of **0.00000000000000000000003g**. We take the same approach as with the larger number and move the decimal point to be between the 1st and 2nd significant digit. However, as we are now moving the decimal place to the right, we need to show this difference in orders of magnitude, this is done using a negative index on the 10.

Standard Form: 0.000000000000000000000003g

Scientific Notation: 3×10^{-23} g

1.10 Recap of Learning Intentions

Use this section to review/comment on how you are feeling regarding the learning intentions.

After this week's lesson you will be able to;

- Differentiate between the various number systems (Natural, Integers etc.)
- Construct line segments with lengths of irrational value
- Represent numbers in scientific notation
- Represent a number correct to a certain number of significant figures

1.10 Recap of Learning Intentions

Tick ✓ for yes or cross ✗ for whether a particular number belongs to the appropriate number system.

Number	N	Q	R	Z	R \ Q
9					
$\sqrt{5}$					
-2					
8.5					
$\frac{2\pi}{5}$					
$\sqrt{16}$					